Covariate Balancing Propensity Score

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Causal Inference Symposium
(ASA New Jersey Chapter)

Joint work with Marc Ratkovic

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- Importance of statistical methods to adjust for confounding factors

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- The main result:

$$\{Y_i(1), Y_i(0)\} \perp\!\!\!\perp T_i \mid \pi(X_i)$$

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- Doubly-robust estimators (Robins *et al.*):

$$\frac{1}{n}\sum_{i=1}^{n}\left[\left\{\hat{\mu}(1,X_i) + \frac{T_i(Y_i - \hat{\mu}(1,X_i))}{\hat{\pi}(X_i)}\right\} - \left\{\hat{\mu}(0,X_i) + \frac{(1-T_i)(Y_i - \hat{\mu}(0,X_i))}{1-\hat{\pi}(X_i)}\right\}\right]$$

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- They have become standard tools for applied researchers

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- Propensity score methods can be sensitive to misspecification

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  4. Doubly-robust least squares regression

Weighting Estimators Do Fine If the Model is Correct

		Bias		**RMSE**	
Sample size	Estimator	GLM	True	GLM	True
(1) Both models correct					
$n = 200$	HT	−0.01	0.68	13.07	23.72
	IPW	−0.09	−0.11	4.01	4.90
	WLS	0.03	0.03	2.57	2.57
	DR	0.03	0.03	2.57	2.57
$n = 1000$	HT	−0.03	0.29	4.86	10.52
	IPW	−0.02	−0.01	1.73	2.25
	WLS	−0.00	−0.00	1.14	1.14
	DR	−0.00	−0.00	1.14	1.14
(2) Propensity score model correct					
$n = 200$	HT	−0.32	−0.17	12.49	23.49
	IPW	−0.27	−0.35	3.94	4.90
	WLS	−0.07	−0.07	2.59	2.59
	DR	−0.07	−0.07	2.59	2.59
$n = 1000$	HT	0.03	0.01	4.93	10.62
	IPW	−0.02	−0.04	1.76	2.26
	WLS	−0.01	−0.01	1.14	1.14
	DR	−0.01	−0.01	1.14	1.14

Weighting Estimators Are Sensitive to Misspecification

		Bias		RMSE	
Sample size	Estimator	GLM	True	GLM	True
(3) Outcome model correct					
$n = 200$	HT	24.72	0.25	141.09	23.76
	IPW	2.69	−0.17	10.51	4.89
	WLS	−1.95	0.49	3.86	3.31
	DR	0.01	0.01	2.62	2.56
$n = 1000$	HT	69.13	−0.10	1329.31	10.36
	IPW	6.20	−0.04	13.74	2.23
	WLS	−2.67	0.18	3.08	1.48
	DR	0.05	0.02	4.86	1.15
(4) Both models incorrect					
$n = 200$	HT	25.88	−0.14	186.53	23.65
	IPW	2.58	−0.24	10.32	4.92
	WLS	−1.96	0.47	3.86	3.31
	DR	−5.69	0.33	39.54	3.69
$n = 1000$	HT	60.60	0.05	1387.53	10.52
	IPW	6.18	−0.04	13.40	2.24
	WLS	−2.68	0.17	3.09	1.47
	DR	−20.20	0.07	615.05	1.75

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  - One-to-one nearest neighbor matching with replacement

Propensity score model	Estimates
Linear	-835
	(886)
Quadratic	-1620
	(1003)
Smith and Todd (2005)	-1910
	(1004)

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  2. Balancing condition:
     - For the Average Treatment Effect (ATE)

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     - For the Average Treatment Effect for the Treated (ATT)

$$\mathbb{E}\left\{T_i \widetilde{X}_i - \frac{\pi_\beta(X_i)(1-T_i)\widetilde{X}_i}{1-\pi_\beta(X_i)}\right\} = 0$$

where $\widetilde{X}_i = f(X_i)$ is any vector-valued function

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- Newton-type optimization algorithm with MLE as starting values

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- Failure to reject the null does not imply the model is correct
- An alternative estimation framework: empirical likelihood

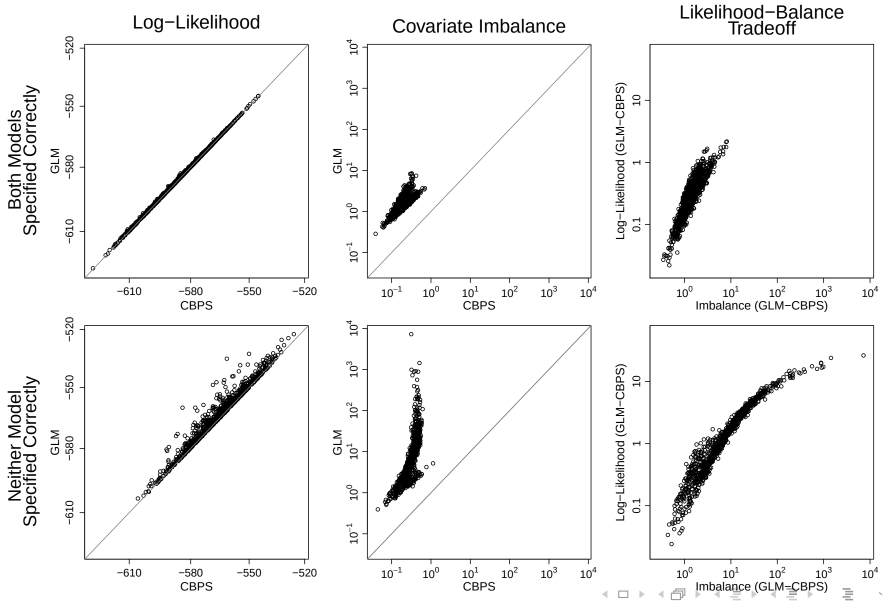
Revisiting Kang and Schafer (2007)

		Bias				RMSE			
Sample size	Estimator	GLM	Balance	CBPS	True	GLM	Balance	CBPS	True
(1) Both models correct									
$n = 200$	HT	-0.01	2.02	0.73	0.68	13.07	4.65	4.04	23.72
	IPW	-0.09	0.05	-0.09	-0.11	4.01	3.23	3.23	4.90
	WLS	0.03	0.03	0.03	0.03	2.57	2.57	2.57	2.57
	DR	0.03	0.03	0.03	0.03	2.57	2.57	2.57	2.57
$n = 1000$	HT	-0.03	0.39	0.15	0.29	4.86	1.77	1.80	10.52
	IPW	-0.02	0.00	-0.03	-0.01	1.73	1.44	1.45	2.25
	WLS	-0.00	-0.00	-0.00	-0.00	1.14	1.14	1.14	1.14
	DR	-0.00	-0.00	-0.00	-0.00	1.14	1.14	1.14	1.14
(2) Propensity score model correct									
$n = 200$	HT	-0.32	1.88	0.55	-0.17	12.49	4.67	4.06	23.49
	IPW	-0.27	-0.12	-0.26	-0.35	3.94	3.26	3.27	4.90
	WLS	-0.07	-0.07	-0.07	-0.07	2.59	2.59	2.59	2.59
	DR	-0.07	-0.07	-0.07	-0.07	2.59	2.59	2.59	2.59
$n = 1000$	HT	0.03	0.38	0.15	0.01	4.93	1.75	1.79	10.62
	IPW	-0.02	-0.00	-0.03	-0.04	1.76	1.45	1.46	2.26
	WLS	-0.01	-0.01	-0.01	-0.01	1.14	1.14	1.14	1.14
	DR	-0.01	-0.01	-0.01	-0.01	1.14	1.14	1.14	1.14

CBPS Makes Weighting Methods Work Better

		Bias				RMSE			
Sample size	Estimator	GLM	Balance	CBPS	True	GLM	Balance	CBPS	True
(3) Outcome model correct									
$n = 200$	HT	24.72	0.33	−0.47	0.25	141.09	4.55	3.70	23.76
	IPW	2.69	−0.71	−0.80	−0.17	10.51	3.50	3.51	4.89
	WLS	−1.95	−2.01	−1.99	0.49	3.86	3.88	3.88	3.31
	DR	0.01	0.01	0.01	0.01	2.62	2.56	2.56	2.56
$n = 1000$	HT	69.13	−2.14	−1.55	−0.10	1329.31	3.12	2.63	10.36
	IPW	6.20	−0.87	−0.73	−0.04	13.74	1.87	1.80	2.23
	WLS	−2.67	−2.68	−2.69	0.18	3.08	3.13	3.14	1.48
	DR	0.05	0.02	0.02	0.02	4.86	1.16	1.16	1.15
(4) Both models incorrect									
$n = 200$	HT	25.88	0.39	−0.41	−0.14	186.53	4.64	3.69	23.65
	IPW	2.58	−0.71	−0.80	−0.24	10.32	3.49	3.50	4.92
	WLS	−1.96	−2.01	−2.00	0.47	3.86	3.88	3.88	3.31
	DR	−5.69	−2.20	−2.18	0.33	39.54	4.22	4.23	3.69
$n = 1000$	HT	60.60	−2.16	−1.56	0.05	1387.53	3.11	2.62	10.52
	IPW	6.18	−0.87	−0.72	−0.04	13.40	1.86	1.80	2.24
	WLS	−2.68	−2.69	−2.70	0.17	3.09	3.14	3.15	1.47
	DR	−20.20	−2.89	−2.94	0.07	615.05	3.47	3.53	1.75

CBPS Sacrifices Likelihood for Better Balance



Revisiting Smith and Todd (2005)

- Evaluation bias: "true" bias $= 0$
- CBPS improves propensity score matching across specifications and matching methods

	1-to-1 Nearest Neighbor			Optimal 1-to-N Nearest Neighbor		
Specification	GLM	Balance	CBPS	GLM	Balance	CBPS
Linear	−835	−559	−302	−885	−257	−38
	(886)	(898)	(873)	(435)	(492)	(488)
Quadratic	−1620	−967	−1040	−1270	−306	−140
	(1003)	(882)	(831)	(406)	(407)	(392)
Smith & Todd	−1910	−1040	−1313	−1029	−672	−32
	(1004)	(860)	(800)	(413)	(387)	(397)

Standardized Covariate Imbalance

- Covariate imbalance in the (Optimal 1–to–N) matched sample

	Linear			Quadratic			Smith & Todd		
	GLM	Balance	CBPS	GLM	Balance	CBPS	GLM	Balance	CBPS
Age	−0.060	−0.035	−0.063	−0.060	−0.035	−0.063	−0.031	0.035	−0.013
Education	−0.208	−0.142	−0.126	−0.208	−0.142	−0.126	−0.262	−0.168	−0.108
Black	−0.087	0.005	−0.022	−0.087	0.005	−0.022	−0.082	−0.032	−0.093
Married	0.145	0.028	0.037	0.145	0.028	0.037	0.171	0.031	0.029
High school	0.133	0.089	0.174	0.133	0.089	0.174	0.189	0.095	0.160
74 earnings	−0.090	0.025	0.039	−0.090	0.025	0.039	−0.079	0.011	0.019
75 earnings	−0.118	0.014	0.043	−0.118	0.014	0.043	−0.120	−0.010	0.041
Hispanic	0.104	−0.013	0.000	0.104	−0.013	0.000	0.061	0.034	0.102
74 employed	0.083	0.051	−0.017	0.083	0.051	−0.017	0.059	0.068	0.022
75 employed	0.073	−0.023	−0.036	0.073	−0.023	−0.036	0.099	−0.027	−0.098
Log-likelihood	−326	−342	−345	−293	−307	−297	−295	−231	−296
Imbalance	0.507	0.264	0.312	0.544	0.304	0.300	0.515	0.359	0.383

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- All of these are situations where balance checking is difficult

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- Balancing moment conditions:

$$\mathbb{E}\left\{\frac{\mathbf{1}\{T_i = k\}\widetilde{X}_i}{\pi_\beta^k(X_i)} - \frac{\mathbf{1}\{T_i = k-1\}\widetilde{X}_i}{\pi_\beta^{k-1}(X_i)}\right\} = 0$$

for each $k = 1, \ldots, K-1$.

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- Balancing between experimental and non-experimental sample:

$$\mathbb{E}\left\{ \frac{S_i \widetilde{X}_i}{\pi_\beta(X_i)} - \frac{(1 - S_i)\widetilde{X}_i}{1 - \pi_\beta(X_i)} \right\} = 0$$

Generalizing Experimental Estimates

- Lack of external validity for experimental estimates
- Target population $\mathcal{P}$
- Experimental sample: $S_i = 1$ with $i = 1, 2, \ldots, N_e$
- Non-experimental sample: $S_i = 0$ with $i = N_e + 1, \ldots, N$
- Sampling on observables: $\{Y_i(1), Y_i(0)\} \perp\!\!\!\perp S_i \mid X_i$
- Propensity score: $\pi_\beta(X_i) = \Pr(S_i \mid X_i)$
- Weighted regression with the weight $= 1/\pi_\beta(X_i)$
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- You may also balance weighted treatment and control groups

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Marginal Structural Models

- Weighted regression of Y_{ij} given $\overline{T}_{ij}$ where the stabilized weight for unit i at time j is given by

$$w_{ij} \quad = \quad \frac{\prod_{j'=1}^{j} \Pr(T_j = T_{ij'} \mid \overline{T}_{j'-1} = \overline{T}_{i,j'-1})}{\prod_{j'=1}^{j} \pi_\beta(\overline{T}_{i,j-1}, \overline{X}_{ij})}$$

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where $\overline{Z}_{ij} = f(\overline{T}_{i,j-1}, \overline{X}_{ij})$

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- Observed and principal strata:

	$Z_i = 1$	$Z_i = 0$
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$T_i = 0$	Defier/Never-taker	Complier/Never-taker

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- Complier average treatment effect or (LATE): $\text{ITT}_c = \text{ITT}/\Pr(\text{compliers})$

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